

# Looking for nonstandard CP violation in $B^\pm \rightarrow D_s^\pm \bar{D}^0 \pi^0$ decays

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## Abstract

We study the possibility of measuring nonstandard CP violation effects through Dalitz plot analysis in  $B^\pm \rightarrow D_s^\pm \bar{D}^0 \pi^0$  decays. The accuracy in the extraction of CP violating phases is analyzed by performing a Monte Carlo simulation of the decays, and the magnitude of possible new physics effects is discussed. It is found that this represents a hopeful scenario for the search of new physics.

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## I Introduction

The origin of CP violation in nature is presently one of the most important open questions in particle physics. Indeed, the main goal of the experiments devoted to the study of  $B$  meson decays is either to confirm the picture offered by the Standard Model (SM) or to provide evidences of CP violation mechanisms originated from new physics. Among the various CP-odd observables in  $B$  physics, attention is mostly concentrated in the “gold-plated” channel  $B \rightarrow J/\Psi K_S$ . According to the SM picture, from the analysis of a time-dependent CP asymmetry observed in these decays it is possible to get a “clean” measurement of  $\sin 2\beta$ , where  $\beta$  is one of the angles of the so-called unitarity triangle [1]. Recent measurements by BELLE and BaBar Collaborations, together with previous results from Aleph, Opal and CDF, lead to the (averaged) value  $\sin 2\beta = 0.734 \pm 0.054$ , which is in good agreement with the constraints imposed by other measured CP-conserving and CP-violating observables [2].

In fact, the common belief is that the SM is nothing but an effective manifestation of some underlying fundamental theory. In this way, all tests of the standard mechanism of CP violation, as well as the exploration of signatures of nonstandard physics, become relevant. One important

characteristic of the SM is that it includes only one source of CP violation, namely a complex phase in the quark mixing matrix  $V_{CKM}$ . In general, since overall phases of transition amplitudes are not observable, one has to deal with interference effects in order to search for measurable CP-violating quantities. Within the SM, there are some specific processes in which the amplitude is either dominated by a single contribution, or in which several contributions are significant, all of them carrying the same weak phase. In these situations, weak SM phases are unobservable, and asymmetries between CP conjugated processes are expected to be vanishingly small. This offers an attractive window to search for evidences of new physics, and is the main motivation for this work.

We show here that three body decays  $B^+ \rightarrow D_s^+ \bar{D}^0 \pi^0$  and  $B^- \rightarrow D_s^- D^0 \pi^0$  provide an interesting scenario to look for such effects. For these processes, the main contributions to the decay amplitude in the SM carry a common weak phase, therefore the measurement of relative CP-violating phases, leading to an asymmetry between  $B^+$  and  $B^-$  decays, would represent a signal of new physics. We discuss here the possibility of performing these measurements by means of a Dalitz plot (DP) fit analysis. In general, three body decays of mesons proceed through intermediate resonant channels, and the DP fit analysis allows a direct experimental access to the amplitudes and phases of the main contributions [3]. The usage of this technique for a clean extraction of CP-odd phases has already been proposed in the literature [6] in relation with other CP-violating observables, more precisely, to get clean measurements of the weak angle  $\gamma$  within the SM. From the experimental point of view, the usage of charged  $B$  mesons has the advantage of avoiding flavor-tagging difficulties. In addition, the processes  $B^\pm \rightarrow D_s^\pm \bar{D}^0 \pi^0$  appear to be statistically favored, in view of their relatively high branching ratios of about 1%.

In order to evaluate the experimental perspectives, we perform a Monte Carlo simulation of the actual processes, applying the DP fit technique to evaluate the error in the extraction of possible CP-violating phases. Then we perform a rough theoretical analysis, discussing the expected magnitude of CP violation effects that could arise beyond the SM. According to our results, the considered channels offer a promising scenario to obtain a clear signature of new physics. In the worst case, the lack of evidences would allow to improve the present bounds on the parameters of the model under consideration.

The paper is organized as follows. In Sect. II we describe the general framework, introducing the CP-violating observables. In Sect. III we detail the DP fit procedure and present the results of our simulations. Sect. IV is devoted to the theoretical discussion of new physics effects, while in Sect. V we summarize our main results.

## II CP-violating phases and Dalitz plot fit technique

In this section we describe how the DP fit technique can be applied to disentangle possible effects of new physics in  $B^\pm \rightarrow D_s^\pm \bar{D}^0 \pi^0$  decays. In principle, these processes are expected to proceed through various intermediate resonances, as well as through a direct, nonresonant channel. The total branching ratio will result from the interference of all these contributions. The Dalitz plot maximum likelihood technique is a powerful tool to get a clean disentanglement of the relevant contributing channels, allowing to measure the ratios between the different partial amplitudes *together with their relative phases*. This can be used to perform a clean extraction of CP-violating phases, avoiding many theoretical uncertainties.

Let us begin by summarizing the main steps of this procedure. More details on these ideas

can be found in Refs. [6]. In general, for a given three body decay, in the DP fitting analysis of experimental data one defines a fitting function  $\mathcal{F}(m_1^2, m_2^2)$ , where  $m_1^2$  and  $m_2^2$  are the usual DP phase space variables. In our case this function can be written as

$$\mathcal{F}_{B \rightarrow D_s D^0 \pi^0}(m_1^2, m_2^2) = |\sum_j \alpha_j e^{i\theta_j} A_j(m_1^2, m_2^2)|^2, \quad (1)$$

where  $m_1^2 = (p_{\pi^0} + p_{D^0})^2$ ,  $m_2^2 = (p_{\pi^0} + p_{D_s})^2$ ,  $A_j$  are definite functions corresponding to each partial channel, and  $\alpha_j$  and  $\theta_j$  are real parameters that emerge as outputs from the fit. The index  $j$  labels the intermediate resonant channels, as well as the nonresonant one. For the resonant channels, the main phase space dependence of the functions  $A_j$  is given by the Breit-Wigner (BW) shape characterizing the resonances, together with definite angular functions which depend on the spin of the corresponding resonant state (we come back to this issues in the next section). The nonresonant decay amplitude is assumed to be constant in most experimental analyses. This fitting technique has proven to be successful to describe e.g. three body decays of  $D$  mesons [7]. In those analyses the phases  $\theta_j$  have been extracted with combined statistical and systematic errors as small as a few degrees, in experiments with a few thousands reconstructed events.

In general, the phases  $\theta_j$  can be written as the sum of a “strong” (CP-conserving) phase  $\delta_j$  and a “weak” (CP-violating) phase  $\varphi_j$ . These cannot be measured separately by a single fit. Nevertheless, comparing the outputs from the CP-conjugated  $B^+$  and  $B^-$  decay experiments one can extract both phases  $\delta_j$  and  $\varphi_j$  simply from

$$\delta_j = \frac{1}{2} (\theta_j^+ + \theta_j^-) \quad (2)$$

$$\varphi_j = \frac{1}{2} (\theta_j^+ - \theta_j^-), \quad (3)$$

where  $\theta_j^+$  ( $\theta_j^-$ ) stands for the phases measured from the  $B^+$  ( $B^-$ ) decays. It is worth to notice that weak phases can be extracted even in the limit where strong phases  $\delta_j$  are vanishingly small—which is expected to be the case in many  $B$  decays, owing to the large  $b$  quark mass. This represents a remarkable advantage with respect to most proposals of measuring CP asymmetries in charged  $B$  decays. In general, in order to get a sizable asymmetry, one requires the presence of strong FSI phases, which introduce a significant theoretical uncertainty. In our case, however, strong phases are already supplied by the resonance widths in the BW functions [9], and no theoretical estimation of FSI phases is needed. Moreover, the latter can be independently obtained from the fit by means of Eq. (2).

It is important to notice that for the fitting procedure to apply, it is necessary that the decay amplitude receives contributions from at least two intermediate channels carrying different CP violating phases. Indeed, an overall phase is physically meaningless, and the DP fit only allows the measurement of relative phases between different channels.

Let us now analyze the case of the decays  $B^\pm \rightarrow D_s^\pm D^0 \pi^0$  in the framework of a theory including physics beyond the SM. For each intermediate channel, it is natural to assume that new physics occurs at a relatively high energy scale, therefore its effects can be decoupled from the resonance BW functions, the angular functions, and other possible form factors in  $A_j(m_1^2, m_2^2)$  arising from low-energy hadronic interactions. Accordingly, the fitting function  $\mathcal{F}$  will be still of the form proposed in Eq. (1), with the same functions  $A_j$ , and now the complex weights  $\alpha_j e^{i\theta_j}$  will include the effects of new physics:

$$\alpha_j^{\pm(exp)} e^{i\theta_j^{\pm(exp)}} = \alpha_j^{SM} e^{i(\delta_j^{SM} \pm \varphi_j^{SM})} + \alpha_j^{NP} e^{i(\delta_j^{NP} \pm \varphi_j^{NP})}. \quad (4)$$

Here the index (*exp*) refers to the experimentally measurable quantities (outputs of the DP fit), whereas *SM* and *NP* denote Standard Model and new physics contributions respectively. The  $\pm$  signs correspond to decays of  $B^\pm$  mesons.

Within the SM, the short-distance effective Hamiltonian relevant for the decays  $B^\pm \rightarrow D_s^\pm D^0 \pi^0$  has been studied in detail [10], including the effects of strong and electroweak penguin operators. The situation is similar as in the “gold-plated” channel  $B \rightarrow J/\Psi K_S$ , in the sense that the main contributions (both tree and penguin) to the effective operators carry the same weak phase. In this way, this phase is expected to factorize, being common to all (resonant and nonresonant) channels contributing to the decay. One could thus conventionally set  $\varphi_j^{SM} = 0$  for all  $j$ , and no CP asymmetry should be observed between  $B^+$  and  $B^-$  decay patterns in absence of new physics<sup>1</sup>. On the contrary, if new physics is present, the situation may be different. To simplify the analysis, let us assume that only two intermediate channels contribute, namely those mediated by resonances  $D^{*0}$  and  $D_s^{*\pm}$ —say channels 1 and 2, respectively. This is a natural assumption, since in fact they are expected to largely dominate the decay (in any case, if other intermediate channels were shown to provide significant contributions, the procedure we describe here can still be followed on the same grounds). In this two-channel case, only the relative phases between both contributions 1 and 2 can be measured. From Eq. (4), one has

$$\theta^{\pm(exp)} \equiv \theta_1^{\pm(exp)} - \theta_2^{\pm(exp)} = \arg \left[ \frac{\alpha_1^{SM} + \alpha_1^{NP} e^{i(\delta_1^{NP} - \delta_1^{SM} \pm \varphi_1^{NP})}}{\alpha_2^{SM} + \alpha_2^{NP} e^{i(\delta_2^{NP} - \delta_2^{SM} \pm \varphi_2^{NP})}} \right] + \delta_1^{SM} - \delta_2^{SM}. \quad (5)$$

The theoretical framework can be simplified by introducing some natural assumptions. First, it is reasonable to think that the resonance hadronization and decay processes—which are governed by strong interactions in the nonperturbative regime—can be disentangled from the effects of new physics, the latter taking place at a high energy scale. In addition, it is usual to assume that strong FSI are the main source for strong phases  $\delta_j$ , since high energy contributions to CP-conserving phases arising from absorptive parts of QCD and electroweak loop diagrams are shown to be suppressed [12]. In this way, for each resonant channel strong phases should factorize out, i.e.  $\delta_j^{NP} = \delta_j^{SM} \equiv \delta_j$ . On the other hand, in most scenarios of new physics—as well as in the SM itself—, CP-violating phases are essentially determined by the flavor content of the quarks entering the diagrams that dominate the  $b$  quark decay. If this is the case, since both resonant states  $D^{*0} D_s$  and  $D^0 D_s^*$  have the same quark content, one expects that the new CP-violating phases obey  $\varphi_1^{NP} = \varphi_2^{NP} \equiv \varphi^{NP}$ , remaining constant along the phase space. Once these assumptions have been taken into account, the measurable complex weights in Eq. (4) can be written as

$$\alpha_j^{(exp)} e^{i\theta_j^{\pm(exp)}} = (\alpha_j^{SM} + \alpha_j^{NP} e^{\pm i\varphi^{NP}}) e^{i\delta_j}, \quad j = 1, 2. \quad (6)$$

We have dropped here the  $\pm$  signs in  $\alpha_j^{(exp)}$ , since the assumption  $\delta_j^{NP} = \delta_j^{SM}$  implies  $\alpha_j^{+(exp)} = \alpha_j^{-(exp)}$ <sup>2</sup>. The expression for the relative phases  $\theta^{\pm(exp)}$  in Eq. (5) simplifies now to

$$\theta^{\pm(exp)} = \arg \left( \frac{\alpha_1^{SM} + \alpha_1^{NP} e^{\pm i\varphi^{NP}}}{\alpha_2^{SM} + \alpha_2^{NP} e^{\pm i\varphi^{NP}}} \right) + \delta_1 - \delta_2. \quad (7)$$

<sup>1</sup>Within the SM, one expects in fact a tiny CP asymmetry in the decay rates. This has been analyzed in Ref. [11] for the process  $B^- \rightarrow D^0 D_s^-$ , where the effect is found to be about 0.2%.

<sup>2</sup>In principle, this last relation could be experimentally checked through the comparison between the fits for  $B^+$  and  $B^-$  decays, providing a consistency test for our assumptions on the strong phases.

As stated, we are interested in the difference between the relative phases for the CP-conjugated decays  $B^+$  and  $B^-$ , which is an observable of CP violation. This is given by

$$\Delta\theta^{(exp)} \equiv \theta^{+(exp)} - \theta^{-(exp)} = \arg\left(\frac{\alpha_1^{SM} + \alpha_1^{NP} e^{+i\varphi^{NP}}}{\alpha_2^{SM} + \alpha_2^{NP} e^{+i\varphi^{NP}}}\right) - \arg\left(\frac{\alpha_1^{SM} + \alpha_1^{NP} e^{-i\varphi^{NP}}}{\alpha_2^{SM} + \alpha_2^{NP} e^{-i\varphi^{NP}}}\right). \quad (8)$$

Finally, assuming that new physics contributions are small when compared to SM amplitudes, i.e.,  $\alpha_j^{NP} \ll \alpha_j^{SM}$ , we end up with

$$\Delta\theta^{(exp)} \simeq 2 \sin \varphi^{NP} \left( \frac{\alpha_1^{NP}}{\alpha_1^{SM}} - \frac{\alpha_2^{NP}}{\alpha_2^{SM}} \right). \quad (9)$$

Notice that this quantity is independent of any CP-conserving phase. It only depends on the (in principle, unknown) real amplitudes  $\alpha_j^{SM}$  and  $\alpha_j^{NP}$ , and on the (also unknown) CP-violating phase  $\varphi^{NP}$ .

As we have discussed above,  $\Delta\theta^{(exp)}$  vanishes in the absence of new physics. This is a convenient situation for the search of clean effects of physics beyond the SM, provided that the factors in the r.h.s. of Eq. (9) are large enough to allow a clear experimental signature. The perspectives in this sense are addressed in the next sections.

### III Experimental perspectives

In this section we make an estimate of the precision that may be reached in the measurement of the phase difference  $\Delta\theta^{(exp)}$  in  $B^\pm \rightarrow D_s^\pm \bar{D}^0 \pi^0$  decays. This will indicate, according to the result in Eq. (9), the minimum size of new physics contributions to the decay amplitudes needed to yield a distinguishable experimental signal.

One important reason for which the channels considered here deserve special attention is their relatively high statistics. Since the branching ratios for  $B^\pm \rightarrow D_s^\pm \bar{D}^0 \pi^0$  are as large as  $\sim 1\%$ , after a couple of years of full run of LHCb, and assuming a 20% reconstruction efficiency, one should end up with some  $10^5$  reconstructed events in each  $B^+$  and  $B^-$  Dalitz plots. This is a large number, taking into account that DP fits performed for  $D$  meson decays with much less events have led to the measurement of relative phases with statistical errors of just a few degrees [3]. However, the processes considered here are very different from those. Indeed, even if such a large number of events will certainly give a very precise measurement of the branching fractions for each partial channel —i.e., the quantities  $\alpha_j^{(exp)}$ —, a precise measurement of phases requires not only large statistics but also a large interference region between the different intermediate channels. It is not obvious that this will be the case for  $B^\pm \rightarrow D_s^\pm \bar{D}^0 \pi^0$  decays, since the involved resonances are very narrow, their widths laying below 1 MeV [8].

In order to evaluate the actual experimental feasibility of our proposal, we have carried out a Monte Carlo simulation of the decays. Our goal is to generate  $10^5$  events in the Dalitz plot, and then to perform a Dalitz plot fit analysis in order to determine if the phases can be successfully extracted with a small statistical error. Clearly, this simulation does not account for the details concerning the detectors. The possible impact of systematic errors will be discussed below.

We have generated  $10^5$  events using a decay amplitude of the form in Eq. (1). As a first guess, we include in the decay only three channels, namely those mediated by the resonances  $\bar{D}^{*0}$  and

$D_s^{*\pm}$ , and the direct nonresonant decay  $B^\pm \rightarrow (D_s^\pm D^0 \pi^0)_{NR}$ . The form of the functions  $A_j$  for the resonances  $j = 1, 2$  is [3]

$$A_j = BW_j(m_j^2) (\vec{p}_B \cdot \vec{p}_\pi) F_j(m_j^2), \quad (10)$$

where the invariant masses  $m_j^2$  are defined as in Eq. (1),  $F_j(m_j^2)$  is a form factor, and  $BW_j(s)$  is the Breit-Wigner function

$$BW_j(s) = \frac{1}{m_{R_j}^2 - s - im_{R_j} \Gamma_{R_j}(s)}, \quad (11)$$

$m_{R_j}$  being the resonance masses ( $R_1 = D^{*0}$ ,  $R_2 = D_s^{*\pm}$ ). For each  $j$ , the  $B$  and  $\pi$  meson three-momenta in Eq. (10) are evaluated in the rest frame of the corresponding intermediate resonance.

We have taken the usual expressions [4] for the form factors<sup>3</sup> and for the momentum-dependent width  $\Gamma_{R_j}(s)$ . The latter is given by

$$\Gamma_{R_j}(s) = \Gamma_{R_j} \frac{m_{R_j}}{\sqrt{s}} \left| \frac{\vec{p}(s)}{\vec{p}(m_{R_j}^2)} \right|^3, \quad (12)$$

where  $\Gamma_{R_j}$  is the on-shell resonance width, and  $\vec{p}(q^2)$  stands for the three-momentum of the resonance decay products when the resonance mass is  $\sqrt{q^2}$ . The shape of the nonresonant decay amplitude, which is in general unknown [13], has been taken —as it is usually done— as a constant function. In any case, as it is discussed below, this assumption has a negligible impact on our results.

In order to carry out the generation of events, we need to introduce as input data the values for the physical quantities  $\alpha_j$ ,  $\theta_j$  and the resonance widths. The expected relative weights  $\alpha_j$  of the two resonant channels can be obtained from the known branching ratios  $BR(B^- \rightarrow D^{*0} D_s^-)$ ,  $BR(D^{*0} \rightarrow D^0 \pi^0)$ ,  $BR(B^- \rightarrow D_s^{*-} D^0)$  and  $BR(D_s^{*-} \rightarrow D_s^- \pi^0)$ . We have [8]

$$\frac{\alpha_1}{\alpha_2} = \sqrt{\frac{BR(B^- \rightarrow D^{*0} D_s^-) \times BR(D^{*0} \rightarrow D^0 \pi^0)}{BR(B^- \rightarrow D_s^{*-} D^0) \times BR(D_s^{*-} \rightarrow D_s^- \pi^0)}} \sim 4. \quad (13)$$

On the other hand, the nonresonant decay amplitude is uncertain; it is just expected to be smaller than the resonant channels. Taking into account that only the relative values between the three coefficients  $\alpha_j$  have a physical meaning in the simulation, we have taken  $\alpha_1 = 1$  and  $\alpha_2 = 0.25$ , while for  $\alpha_{NR}$  we have considered different values, ranging from 0 to 0.1. Concerning the phases  $\theta_j$ , one expects CP-conserving parts to be relatively small, whereas the CP-violating SM phase is essentially the same for all amplitudes and can be factorized out. Thus, assuming that SM contributions dominate, it is reasonable to choose all phases  $\theta_j$  in our numerical simulation to be small numbers. In fact, it will be seen that this assumption is not relevant to our conclusions.

Finally, other relevant inputs in our simulation are the on-shell widths  $\Gamma_{R_j}$  of both resonances: events coming from a narrow resonance should be concentrated in a given region of the plot, hence the interference region between both resonances is expected to be relatively small. Present measurements of  $D^{*0}$  and  $D_s^{*\pm}$  widths are not conclusive, giving in both cases only upper bounds of about 2 MeV. For our simulations, we have chosen to consider values ranging from 0.01 to 1

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<sup>3</sup>The actual shape of the form factors in  $B$  decays is in general unknown. We have considered expressions similar to those used for  $D$  decays, finding that their incidence is not relevant to the discussion in this work.

MeV. The recent measurement of the  $D^{*+}$  width, which is found to be around 0.1 MeV [14], can be thought as a hint of the expected orders of magnitude.

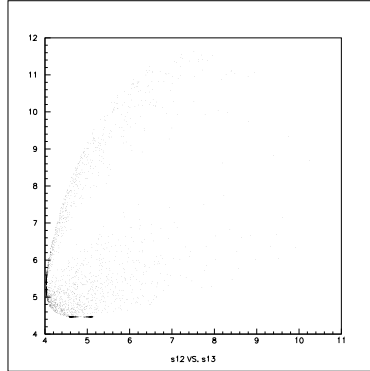


Figure 1: Dalitz plot for the  $B^- \rightarrow D_s^- D^0 \pi^0$  decay

As an example, we show in Fig. 1 the Dalitz plot generated with  $\alpha_{NR} = 0.1$ ,  $\theta_1 = 0$ ,  $\theta_2 = 20^\circ$ ,  $\theta_{NR} = 10^\circ$ , and equal widths of 1 MeV for both resonances (besides the already given values of  $\alpha_1 = 1$  and  $\alpha_2 = 0.25$ ). One observes that, even if both resonances are quite narrow, the events appear to be spread out in a large region of the plot. This is the consequence of a purely kinematic effect, due to the fact that both resonances are located very close to the threshold of the phase space. This effect compensates the narrow width suppression, and brings a good hope to extract the relative phases successfully.

After carrying out this simulation of the decay, we have performed a fit of the data according to the fitting function given in Eq. (1), where now the coefficients  $\alpha_j$  and  $\theta_j$  are left as free parameters. In fact, as explained above, the fit provides only *relative* values for both amplitudes and phases [3], therefore we have kept fixed the reference values  $\alpha_1 = 1$  and  $\theta_1 = 0$ . The result of the fit is given in Table 1. The method allows to extract the phase  $\theta_2$  with a statistical error as small as  $1.4^\circ$ .

channel	$\alpha_j$	$\theta_j$
$D^{*0} D_s$	fixed	fixed
$D^0 D_s^*$	$0.2514 \pm 0.0017$	$(20.7 \pm 1.4)^\circ$
nonresonant	$0.1007 \pm 0.0020$	$(9.1 \pm 1.2)^\circ$

Table 1: Fitting results of the Monte Carlo sample. The events have been generated with  $\alpha_2 = 0.25$ ,  $\theta_2 = 20^\circ$ ,  $\alpha_{NR} = 0.1$ ,  $\theta_{NR} = 10^\circ$ , and  $\Gamma_{D^{*0}} = \Gamma_{D_s^*} = 1$  MeV.

We have performed a systematic study of the results of the fit allowing reasonable ranges of variation for the unknown quantities used to generate the Monte Carlo sample, namely the resonance widths, the weight  $\alpha_{NR}$  and the relative phases  $\theta_j$ . As a first outcome of this analysis, it is found that the statistical errors are independent of the initial values of the phases. Secondly, the errors for both the extracted amplitude and phase of the  $D_s^*$  mediated decay (channel 2) are independent of the weight  $\alpha_{NR}$  of the nonresonant channel, *even in the limit*  $\alpha_{NR} = 0$ . This shows that the interference between the two resonant channels is not mediated by the nonresonant one, but arises from the above mentioned spread out of the events corresponding to resonance-mediated

decays. Finally, as expected, it is found that the errors in the extracted weights  $\alpha_j$  are independent of the resonance widths; on the contrary, the values of the widths do affect the quantity we are interested in, i.e. the error in the extracted relative phase  $\theta_2 - \theta_1$ . This dependence is illustrated by the results in Table 2, where we have considered several simulations in which the amplitudes and phases  $\alpha_j$ ,  $\theta_j$  have been taken as in the previously described example. We quote in the Table the errors obtained in the extraction of  $\theta_2 - \theta_1$  for different values of  $\Gamma_{D^{*0}}$  and  $\Gamma_{D_s^*}$ . In the first five rows of the Table we have assumed equal  $D^{*0}$  and  $D_s^*$  widths, while in the last row we have taken  $\Gamma_{D^{*0}} = 100$  KeV,  $\Gamma_{D_s^*} = 10$  KeV (in fact, a relative suppression of the  $D_s^*$  width could be expected since the strong decay  $D_s^{*\pm} \rightarrow D_s^\pm \pi^0$  violates isospin). We see here that for a width as narrow as 10 KeV the phase difference can still be extracted with relatively low statistical error.

$\Gamma_{D^{*0}} ; \Gamma_{D_s^*}$ (MeV)	Error
1	1.4°
0.5	1.5°
0.1	1.7°
0.05	2.3°
0.02	5.1°
0.1 ; 0.01	4.0°

Table 2: Errors in the extracted value of  $\theta_2 - \theta_1$ , for different values of resonance widths. Input amplitudes and phases for the event generation are same as in Table I.

Before ending this section let us say a few words about systematic (experimental) errors in the extraction of phases. The evaluation of these errors is in general a quite difficult task. In order to carry out the complete analysis, one should perform a full numerical simulation of the experiment including the detector, which is out of the scope of this paper. Nevertheless, in order to have an estimate we can take into account the results from recent DP analyses [4, 5]. The latter suggest that the systematic error in the measurement of phases for intermediate channels with large branching fractions should not be above a few degrees, i.e. of the same order of those quoted in Table 2.

## IV Expected size of new physics effects

Let us now turn back to Eq. (9) and analyze the theoretical expectations for the size of  $\Delta\theta^{(exp)}$  in the context of a theory beyond the SM, in order to evaluate if this observable has potential chances to provide experimental evidences of new physics. To carry out the theoretical analysis we take into account the low-energy effective Hamiltonian relevant for the processes under consideration, including QCD corrections at the leading order. Then, to deal with long-range matrix elements, we use the simple factorization approach [15], which should be adequate to estimate the significance of the new contributions [16].

In view of the large hadronic uncertainties and the usual amount of freedom to fix new physics parameters, we do not intend to perform an accurate calculation of possible nonstandard contributions to the  $B^\pm \rightarrow D_s^\pm D^0 \pi^0$  decay amplitude. Just as an illustrative example, we consider the rather representative framework of multihiggs models, showing that the situation becomes quite promising if nonstandard contributions to penguin diagrams are comparable to those arising from



SM physics.

Our theoretical analysis is based on the  $\Delta B = 1$  effective Hamiltonian [10, 17]

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \left\{ V_{cb} V_{cs}^* (C_1 O_1 + C_2 O_2) - V_{tb} V_{ts}^* \left( \sum_{i=3}^7 C_i O_i \right) \right\}, \quad (14)$$

where  $C_i$  are Wilson coefficients evaluated at a renormalization scale  $\mu \approx m_b$ , and  $O_i$  are local operators,

$$\begin{aligned} O_1 &= (\bar{c}_\alpha b_\alpha)_{V-A} (\bar{s}_\beta c_\beta)_{V-A} & O_2 &= (\bar{c}_\beta b_\alpha)_{V-A} (\bar{s}_\alpha c_\beta)_{V-A} \\ O_3 &= (\bar{s}_\alpha b_\alpha)_{V-A} \sum_{q'} (\bar{q}'_\beta q'_\beta)_{V-A} & O_4 &= (\bar{s}_\beta b_\alpha)_{V-A} \sum_{q'} (\bar{q}'_\alpha q'_\beta)_{V-A} \\ O_5 &= (\bar{s}_\alpha b_\alpha)_{V-A} \sum_{q'} (\bar{q}'_\beta q'_\beta)_{V+A} & O_6 &= (\bar{s}_\beta b_\alpha)_{V-A} \sum_{q'} (\bar{q}'_\alpha q'_\beta)_{V+A} \\ O_7 &= (g_s/8\pi^2) m_b \bar{s}_\alpha \sigma^{\mu\nu} (1 + \gamma_5) T_{\alpha\beta}^a b_\beta G_{\mu\nu}^a. \end{aligned} \quad (15)$$

Here  $V \pm A$  refers to the Lorentz structure  $\gamma_\mu(1 \pm \gamma_5)$ ,  $\alpha$  and  $\beta$  stand for  $SU(3)$  color indices,  $T_{\alpha\beta}^a$  are generators of  $SU(3)$  color transformations and  $G_{\mu\nu}^a$  denotes the gluonic field strength tensor. Contributions from electroweak penguins will not be taken into account, therefore these operators have not been included in (14). We will also neglect the effect of the electromagnetic dipole operator. Within the SM, the coefficients  $C_i$  can be calculated at the scale  $m_W$ , and then evolved to  $\mu \approx m_b$  through the renormalization group equations [10]. The  $V_{CKM}$  factors corresponding to each operator have been explicitly separated in (14), so that with good approximation the coefficients  $C_i$  in the SM can be assumed to be real numbers<sup>4</sup>. Moreover, in view of the unitarity of the  $V_{CKM}$  matrix, one has  $V_{tb} V_{ts}^* = -V_{cb} V_{cs}^* - V_{ub} V_{us}^* \simeq -V_{cb} V_{cs}^*$ , where the correction due to the  $V_{ub} V_{us}^*$  term is about 5%. In this way, for the case under consideration, the CP-violating phase carried by the penguin contributions in the SM is approximately the same as that coming from the tree operators  $C_1$  and  $C_2$ , and will factorize out for the decay amplitudes of interest (the contribution of the  $V_{ub} V_{us}^*$  term to the full amplitude will be below 0.5% if, as expected, the total penguin amplitude does not exceed 10% of the tree piece). In a given extension of the SM, however, the coefficients  $C_i$  will carry in general nonvanishing CP-violating phases  $\varphi_i$ , allowing for the interference effects discussed in the previous sections.

In general, in a theory including physics beyond the SM, one expects that the new particles can be integrated out at the  $m_W$  scale, leading to new contributions to the coefficients  $C_i(m_W)$ . However, since the new particles have been integrated out, the running of the coefficients down to  $\mu \approx m_b$  proceeds just as in the SM [18]. This running of SM coefficients has been analyzed in detail in Refs. [10, 19] and will not be repeated here.

In the evaluation of the amplitudes  $\langle VP | \mathcal{H}_{\text{eff}} | B \rangle$ , the scale and renormalization scheme dependence introduced by the coefficients  $C_i$  should be compensated by that of the matrix elements of the quark operators  $O_i$  between the hadronic states. However, as stated above, to evaluate these quantities we will use the factorization ansatz, and in this approach the matrix elements are written in terms of decay constants and form factors, which are both scale and renormalization scheme independent. In order to achieve the required cancellation, it is possible [17] to calculate the one-loop corrections to the partonic matrix elements  $\langle s\bar{c}c | O_i | b \rangle$ , and to define new effective coefficients  $C_i^{\text{eff}}$

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<sup>4</sup>In fact, they carry small CP-violating and CP-conserving phases, coming from Cabibbo-suppressed contributions and absorptive parts of loop diagrams respectively.

such that the one-loop quark-level matrix elements read

$$\langle s\bar{c}c|\mathcal{H}_{\text{eff}}|b\rangle = \sum_{i=1}^6 C_i^{\text{eff}} \langle s\bar{c}c|O_i|b\rangle^{\text{tree}}. \quad (16)$$

At NLO these effective coefficients will be given by the original  $C_i$  plus QCD corrections,

$$C_i^{\text{eff}} = C_i(\mu) + \frac{\alpha_s}{4\pi} \sum_{j=1}^7 K_{ij}(\mu) C_j(\mu). \quad (17)$$

The analytic expressions for the functions  $K_{ij}$  can be found in Refs. [17, 20, 21]. It can be shown that now the effective coefficients  $C_i^{\text{eff}}$  are scale and scheme independent, as well as gauge invariant and infrared safe [22]. An important point is that the corrections introduced in Eq. (17) involve the coefficient  $C_7$ , which can receive important contributions coming from nonstandard physics, as occurs e.g. in the case of two-Higgs-doublet models [18]. Even if the operator  $O_7$  does not contribute directly to the  $B \rightarrow VP$  decay amplitudes in the factorization approach, the combination in (17) implies that the new physics corrections to  $C_7$  are translated to other effective coefficients  $C_i^{\text{eff}}$  and thus to the decay amplitude.

The previous analysis can be now applied to the decays of our interest, namely the resonant processes  $B^- \rightarrow D_s^{*-} D^0$ ;  $D_s^{*-} \rightarrow D_s^- \pi^0$  and  $B^- \rightarrow D^{*0} D_s^-$ ;  $D^{*0} \rightarrow D^0 \pi^0$  that dominate the three body decay  $B^- \rightarrow D^0 D_s^- \pi^0$ . In the described framework, the relevant two-body amplitudes  $\langle D_s^{*-} D^0 | \mathcal{H}_{\text{eff}} | B^- \rangle$  and  $\langle D^{*0} D_s^- | \mathcal{H}_{\text{eff}} | B^- \rangle$  will be given by

$$\begin{aligned} \langle VP | \mathcal{H}_{\text{eff}} | B^- \rangle &= \frac{G_F}{\sqrt{2}} V_{cb} V_{cs}^* \sum_{i=1}^6 C_i^{\text{eff}} \langle VP | O_i | B^- \rangle_{FA} \\ &= \frac{G_F}{\sqrt{2}} V_{cb} V_{cs}^* \tilde{a}(B^- \rightarrow VP) X^{(B^- \rightarrow VP)}, \end{aligned} \quad (18)$$

where the subindex  $FA$  denotes that the matrix element is evaluated within the factorization approximation. The factor  $\tilde{a}(B^- \rightarrow VP)$  includes the effective coefficients  $C_i^{\text{eff}}$ , whereas  $X^{(B^- \rightarrow VP)}$  contains the form factors related to the factorized amplitudes. For the processes under consideration one has [23, 24]

$$\begin{aligned} \tilde{a}(B^- \rightarrow D^{*0} D_s^-) &= a_1 + a_4 - 2a_6 \frac{m_{D_s}^2}{(m_b + m_c)(m_s + m_c)} \\ \tilde{a}(B^- \rightarrow D_s^{*-} D^0) &= a_1 + a_4, \end{aligned} \quad (19)$$

where the coefficients  $a_i$  are defined as  $a_i \equiv C_i^{\text{eff}} + C_{i+1}^{\text{eff}} / (N_c^{\text{eff}})_i$  for  $i = 1$ , and  $a_i \equiv C_i^{\text{eff}} + C_{i-1}^{\text{eff}} / (N_c^{\text{eff}})_i$  for  $i = 4, 6$ . The effective parameters  $(N_c^{\text{eff}})_i$  in these expressions account for the uncertainties introduced when calculating the matrix elements of the effective operators between hadron states [17, 20, 21]. The factors  $X^{(B^- \rightarrow VP)}$  are given by

$$\begin{aligned} X^{(B^- \rightarrow D^{*0} D_s^-)} &= 2 f_{D_s} m_{D^{*0}} A_0^{BD*}(m_{D_s}^2) (\varepsilon_{D^{*0}}^* \cdot P_B) \\ X^{(B^- \rightarrow D_s^{*-} D^0)} &= 2 f_{D_s^*} m_{D_s^*} F_1^{BD}(m_{D_s^*}^2) (\varepsilon_{D_s^*}^* \cdot P_B), \end{aligned} \quad (20)$$

where  $\varepsilon_V$  are the corresponding  $V$  meson polarizations,  $P_B$  is the  $B$  four-momentum, and the expressions include decay constants and form factors that can be estimated in specific models. In fact, Eqs. (20) have been quoted only for completeness, since the factors  $X^{(B^- \rightarrow VP)}$  cancel out in our estimation for  $\Delta\theta^{(exp)}$ . This can be seen by noticing that the expression for  $\Delta\theta^{(exp)}$  in (9) involves ratios between SM and new physics amplitudes. According to previous assumptions, the effects of new physics are only present in the effective coefficients  $C_i^{\text{eff}}$ —or, equivalently,  $\tilde{a}(B \rightarrow VP)$ —, therefore any global factor will cancel. One has in this way

$$\begin{aligned} \frac{\alpha_1^{NP} e^{-i\varphi^{NP}}}{\alpha_1^{SM}} &= \frac{\langle D^{*0} D_s^- | \mathcal{H}_{\text{eff}} | B^- \rangle^{NP}}{\langle D^{*0} D_s^- | \mathcal{H}_{\text{eff}} | B^- \rangle^{SM}} \simeq \frac{(a_1 + a_4 - 2r a_6)^{NP}}{(a_1 + a_4 - 2r a_6)^{SM}} \\ \frac{\alpha_2^{NP} e^{-i\varphi^{NP}}}{\alpha_2^{SM}} &= \frac{\langle D_s^{*-} D^0 | \mathcal{H}_{\text{eff}} | B^- \rangle^{NP}}{\langle D_s^{*-} D^0 | \mathcal{H}_{\text{eff}} | B^- \rangle^{SM}} \simeq \frac{(a_1 + a_4)^{NP}}{(a_1 + a_4)^{SM}}, \end{aligned} \quad (21)$$

where  $r$  stand for the mass ratio  $m_{D_s}^2 / [(m_b + m_c)(m_s + m_c)]$ , and—as in the previous sections— we have assigned labels 1 and 2 to the channels mediated by the resonances  $D^{*0}$  and  $D_s^*$  respectively. Average values of quark masses yield  $r \simeq 0.5$ .

In order to analyze the possible NP effects in our observable  $\Delta\theta^{(exp)}$ , let us consider the typical situation of a theory including an extended scalar sector. In the case of multihiggs (MH) models, the scalar-mediated tree contributions to  $C_1$  and  $C_2$  can be neglected, since in general scalar couplings are proportional to the current quark masses of the involved vertices. On the other hand, penguin-like diagrams mediated by the new scalars involve vertices which are proportional to the top quark mass, thus they are potentially important. Then, while SM amplitudes are dominated by tree contributions ( $C_{1,2}^{SM} \gg C_i^{SM}$  for  $i = 3 \dots 6$ ), in a MH scheme the main effect of the extended scalar sector on  $\alpha_1$  and  $\alpha_2$  occurs through the new contributions to the effective coefficients  $a_4$  and  $a_6$ . In this way, from Eqs. (9) and (21) one gets

$$\Delta\theta^{(exp)} \simeq 2 \sin \varphi^{MH} \left( \frac{\alpha_1^{MH}}{\alpha_1^{SM}} - \frac{\alpha_2^{MH}}{\alpha_2^{SM}} \right) \sim -4r \sin \varphi^{MH} \frac{|a_6^{MH}|}{a_1}. \quad (22)$$

As a first outcome from this expression, it is seen that the ratios  $\alpha_j^{NP}/\alpha_j^{SM}$  do not cancel with each other, consequently the asymmetry  $\Delta\theta^{(exp)}$  is in principle nonzero.

Even if the result in (22) is just an estimate, it can be taken into account in order to show that new physics effects can be significant enough to provide an observable signal. According to the analysis presented in the previous section, this would be achieved if new physics contributions to  $a_6$  reach about 10% of the SM tree amplitude, and carry a CP-violating phase of order one (one would obtain in this case an asymmetry  $\Delta\theta^{(exp)}$  of about 10 degrees). Within the SM, the effective coefficients  $|a_1|$  (tree) and  $|a_6|$  (penguin) are estimated to be approximately 1 and 0.06, respectively [23]. Thus, one would have important chances of measuring nonstandard physics if new contributions to  $a_6$  carrying large CP-violating phases are comparable in size to SM ones. It is worth to point out that this level of contribution of nonstandard physics is indeed suggested by some puzzling experimental results on penguin-dominated modes, such as the  $B \rightarrow \eta' K$  branching ratios [25] and the time-dependent CP asymmetries in  $B^0 \rightarrow \phi K_S$  [26]. The experimental values for these observables are at least  $2\sigma$  away from SM expectations, and can be seen as indications of large new physics effects at the penguin level.

We believe that these experimental observations on penguin-dominated  $B$  decay channels already provide a substantial ground to encourage the DP analysis of  $B^\pm \rightarrow D_s^\pm D^0 \pi^0$  proposed here.

On the other hand, we point out that the room for nonstandard contributions to penguin amplitudes —and thus to the phase difference  $\Delta\theta^{(exp)}$ — is relatively large, mainly due to the existing theoretical uncertainties in the evaluation of SM amplitudes, and to the large number of unknown parameters included in most scenarios of new physics. To be definite, let us take here as an example one of the simplest possible extensions of the SM, namely a two-Higgs-doublet model (THDM) type III. In particular, we consider a minimal scenario [27] which does not include tree level FCNC, and the number of new parameters is reduced to four (three Yukawa couplings parameters plus the charged Higgs mass). In this framework the main new contributions to  $b$  quark decays arise from one-loop diagrams involving a virtual top quark, while neutral Higgs-mediated diagrams are shown to be negligible [18, 28]. As stated, in this kind of models the largest new contributions to the amplitudes  $a_i$  come through the dipole coefficient  $C_7$ , and the allowed space for the new parameters is mainly constrained by the effects on  $B \rightarrow X_s \gamma$  decays [28]. Taking into account the bounds in Refs. [27, 28], it is possible to estimate the allowed values for both the amplitude  $a_6$  and the CP-violating phase  $\varphi$ . We find that within this model the phase difference  $\Delta\theta^{(exp)}$  can be as large as 3 degrees, which, according to the analysis Sect. III, would be around the limit of observability for the number of events considered.

The example below should be taken just as an illustration to show the potentiality of our analysis through a simple manageable case. Clearly, the inclusion of more degrees of freedom would relax the experimental bounds on the new model parameters (imposed e.g. by the chosen mechanism to avoid unwanted flavor changing neutral transitions), allowing higher values for the measurable phase difference  $\Delta\theta^{(exp)}$  which will exceed the observability limits. In addition, other possible frameworks of nonstandard physics have been shown to provide enhancement effects on penguin-dominated processes, offering an explanation for the puzzling time-dependent CP asymmetries in  $B^0 \rightarrow \phi K_S$ . Among the most popular scenarios, recent analyses include R-parity violating supersymmetry [29], left-right supersymmetric models [30], and theories including warped extra dimensions [31]. In all these models—which include in general a rather large number of new parameters—it has been shown that new physics contributions can be of the same order as SM penguin amplitudes. In this way, their effects on the  $b \rightarrow c\bar{c}s$  channel could provide an observable signal in the DP analysis of  $B^\pm \rightarrow D_s^\pm \bar{D}^{(*)0} \pi^0$  decays proposed here.

## V Summary

We discuss the possible measurement of nonstandard CP violation in  $B^\pm \rightarrow D_s^\pm \bar{D}^{(*)0} \pi^0$ , exploiting the fact that for these processes the asymmetry between  $B^+$  and  $B^-$  decays is expected to be negligibly small in the Standard Model. The presence of two resonant channels provides the necessary interference to allow for CP asymmetries in the differential decay width, even in the limit of vanishing strong rescattering phases.

In order to measure the CP-odd phases entering the interfering contributions to the total decay amplitude, we propose to use the Dalitz Plot fit technique. This allows a clean disentanglement of relative phases, independent of theoretical uncertainties arising from FSI effects. The expected quality of the experimental measurements has been estimated by means of a Monte Carlo simulation of the decays, from which we conclude that the phases can be extracted with a statistical error not larger than a couple of degrees, provided that the widths of the intermediate  $D^{*0}$  and  $D_s^*$  resonances are at least of the order of a hundred keV. On the theoretical side, within the framework

of generalized factorization we perform a rough estimation of possible nonstandard CP violation effects on the interfering amplitudes. We take as an example the typical case of a multihiggs model, showing that the level of accuracy of the DP fit measurements can be sufficient to reveal effects of new physics.

Let us finally stress that tree-dominated decays like  $B^\pm \rightarrow D_s^\pm \bar{D}^0 \pi^0$  are usually not regarded as good candidates to reveal new physics, since the effects on branching ratios are not expected to be strong enough to be separated from the theoretical errors. Our proposal represents a possible way of detecting these effects by means of CP asymmetries, which can allow the disentanglement of new physics contributions to penguin-like operators in a theoretically simple way.

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